HIV/AIDS, social security and the two-tier structure of African economies

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Abstract

This article is on the occurrence of HIV in Africa and its implications on both old-age support schemes and the two-tier economic structure in developing countries. The aim is to clarify some commonly misperceived issues and to draw a basic framework for social security in Africa by emphasizing where the challenge actually lies in how the situation social security is facing in Africa is very different from that in Europe. We model a two-sector economy in a three-period overlapping generations framework where social security exists both formally as well as informally - thus capturing in essence the situation of many African economies. In such setup, we investigate how the occurrence of HIV/AIDS will affect migration between the two sector, where both social security systems act as the motivation to migrate. Calibrating the model on African economies, we investigate how the incidence rate of HIV/AIDS amongst different cohorts leads to different migration equilibria.

JEL Classification: J14, H31

Keywords: Social security, Demographics, Development, HIV/AIDS

1 Introduction

In most low-income developing countries, the time is not ripe for the formulation of a comprehensive social security policy, because so little is known about the opportunities for social insurance in the informal sector (Van Ginneken, 1999).

Africa is currently confronting an HIV/AIDS crisis. For one, the continent is faced with the human tragedy of having such a large proportion of the population being infected: at the end of 2000, an estimated 25.3 million people were infected in Sub-Saharan Africa (SSA) alone, representing approx. 70% of all cases world-wide. HIV/AIDS is now the main cause of death in Africa, overtaking malaria; the adult prevalence rate for SSA is approximately 8.8%\textsuperscript{1}.

Numerous studies have attempted to evaluate the impact of the disease on African economies, focusing mainly on its macroeconomic effects. Results have indicated that HIV/AIDS has a negative (but small) impact on GDP and GDP per capita growth. Thus e.g. Bloom and Mahal (1995) sustain that the prediction that HIV/AIDS will have a major economic effect are not

\textsuperscript{1}Keeping in mind however that averages of course hide discrepancies, one can note that in South Africa and Botswana adult prevalence rates are already around 20% and 36% respectively, whereas in Benin and Guinea-Bissau the prevalence rate was (at end 1997) 1.8% and 1.7% respectively.
borne out by empirical evidence: using an econometric model, estimated on a sample of 51 countries for the period 1980-92, they assess that the HIV/AIDS impact is less significant than that of other standard growth model elements. Kambou et al. (1992), using a CGE approach for Cameroon, and that HIV/AIDS may reduce GDP growth by almost 3 percent over the simulation period. Using the same approach for South Africa, Arndt and Lewis (2000) point out that GDP growth rate might be reduced by about 2.6 percent over a simulation period of 12 years. Cuddington (1993), using data from Tanzania, ands that HIV/AIDS has reduced the average real GDP growth rate by 0.6-0.9 percent.

However, despite this relatively small macroeconomic impact, HIV/AIDS represents a serious source of concern at all levels. By causing mortality and morbidity to rise, it will have serious impacts through changes in the structure of the population and the labour, the reduction of productivity of infected workers, the reduction of disposal income of households affected, the increase of expenditures on health care, capacity building and sick-payment, etc.

This paper seeks to investigate the implication of the disease on old-age support and on the two-tier structure in African economies. We model a two sectors economy in a three-period overlapping generations framework where social security exits both formally as well as informally. In each sector, agents are split into two types - healthy agents, i.e. those who have not contracted HIV/AIDS, and unhealthy ones i.e. those who have. Arguing that social security and increased trust in public institutions act as motivation to migrate, we investigate how the occurrence of HIV/AIDS will affect migration between the two sectors. More precisely, we show that HIV/AIDS impacts on the strength of family ties. It is likely to reduce the return to familial old-age support arrangements and to increase the relative confidence in public announcements. A massive or a progressive migration from the informal sector to the formal sector can be obtained under reasonable conditions. This paper provides theoretical and numerical predictions on the incidence of HIV/AIDS. Distinguishing the cases of a permanent and a temporary shock, it derives conditions under which large migration occurs and the conditions under which a progressive development of the formal sector can be obtained.

The paper is structured as follows. Section 2 provides some stylized facts on HIV/AIDS in Africa and on the African economies based on recent studies and compiled data. Section 3 introduces the model and the different types of migration equilibrium. Section 4 and 5 analyze respectively the implications of a permanent and temporary HIV-shock. Section 6 concludes and discusses policy implications.

2 Stylized Facts

Let us start with some important stylized facts regarding the demographic and economic impacts of HIV/AIDS, the structure of African economies and the old-age support systems.

Global demographic impact of HIV/AIDS. A striking impact of HIV/AIDS in Africa is that almost everyone who is infected will die owing to the lack of treatment; (9 out of 10 persons - Unpd, 2000). Life-expectancy has been reduced by an average of 6.5 years in the 35 most affected countries (see table 1): in Botswana, Namibia, South Africa, Zambia and Zimbabwe the figure will be between 20 and 29 years lower than what it would have been in the absence
of the disease.\textsuperscript{2} The consequences for the 29 African countries with the highest prevalence rates are such that their total population is expected to be 50 million fewer than in the absence of the disease. The populations of Botswana, Namibia and Zimbabwe are projected to be about 20 percent smaller than they would have been without HIV/AIDS; the disease has raised the crude death rate of the 29 countries by 3.4 points between 1995 and 2000, projections indicate that this death increase is going to be about 5 points between 2000 and 2010. In total, approximately 44 million additional deaths due to HIV/AIDS are expected to occur in these countries between 1995 and 2015.

Impact on the age structure. The impact of HIV/AIDS is not equally distributed among the age-groups. Obviously, given the transmission mechanism of the virus, the prevalence rate of adults is expected to be higher. Thus, whilst the average sero-prevalence rate of 33 Sub Saharan African countries (World Bank, 2002) for adults (aged 15-64) was approximately 11%, that for children was approximately 0.43% (Figure 6 gives a scatterplot of sero-prevalence rates - one can observe a clear relationship between adults' prevalence and children's prevalence). However, even amongst adults is the impact varied, the greatest impact occurring in the age group 35-49 (Unpd, 2000). The crude death rate in this age group is projected to rise by almost 8 points between 1995 and 2015 compared with what it would have been in the absence of HIV/AIDS. Moreover, in the 9 countries with the highest HIV/AIDS prevalence rate, the death rate of this age group is estimated to be 21.88 percent versus 9.20 percent if there was not the disease. The second group largely affected is the age group 15-34: 28 percent of the additional deaths in the 29 African countries will affect this group. The ILO points out that the mean age of death among workers in seven enterprises studied in Tanzania is between 31 and 37 years.

HIV/AIDS and the labor market. By affecting particularly these age groups, the HIV/AIDS epidemic has significant social and economic consequences. For one, it affects the labor force by shrinking its size.\textsuperscript{3} It also however jeopardizes its quality and performance, and

\textsuperscript{2} Being 'highly affected' means having an HIV prevalence rate of 2% or more, United Nations Population Division (2001).

\textsuperscript{3} ILO projections indicate that the size of the labor force in these countries will be between 10 and 30 percent smaller by 2020 than it would have been without AIDS. Botswana is expected to lose in 2005 and 2020 about 17.2 and 31 percent of its labor force, Zimbabwe 20 and 29 percent, and Namibia 13 and 35 percent.
Table 1: Expectation of life at birth with and without HIV/AIDS for the most affected countries and groups of affected countries in Africa.

<table>
<thead>
<tr>
<th>Country</th>
<th>1995-2000 with w/o</th>
<th>2000-2005 with w/o</th>
<th>2005-2010 with w/o</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angola</td>
<td>44.6 46.5</td>
<td>45.8 48.5</td>
<td>49.0 52.5</td>
</tr>
<tr>
<td>Benin</td>
<td>53.5 55.0</td>
<td>54.0 57.0</td>
<td>56.0 61.0</td>
</tr>
<tr>
<td>Botswana</td>
<td>44.4 67.6</td>
<td>36.1 69.7</td>
<td>43.0 73.0</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>45.3 53.4</td>
<td>48.1 55.9</td>
<td>56.4 60.9</td>
</tr>
<tr>
<td>Burundi</td>
<td>40.6 49.6</td>
<td>40.6 51.6</td>
<td>44.9 55.5</td>
</tr>
<tr>
<td>Cameroon</td>
<td>50.0 56.2</td>
<td>50.0 58.6</td>
<td>52.9 63.7</td>
</tr>
<tr>
<td>Centr. African R.</td>
<td>44.3 52.9</td>
<td>44.3 55.2</td>
<td>49.1 59.3</td>
</tr>
<tr>
<td>Chad</td>
<td>45.2 47.2</td>
<td>46.3 48.8</td>
<td>50.9 53.4</td>
</tr>
<tr>
<td>Congo</td>
<td>50.9 56.9</td>
<td>51.6 58.9</td>
<td>55.7 63.0</td>
</tr>
<tr>
<td>Cote d'Ivoire</td>
<td>47.7 56.6</td>
<td>47.9 58.6</td>
<td>52.4 62.7</td>
</tr>
<tr>
<td>DRC</td>
<td>50.5 54.9</td>
<td>52.1 58.2</td>
<td>56.3 62.1</td>
</tr>
<tr>
<td>Djibouti</td>
<td>45.5 50.5</td>
<td>40.6 52.4</td>
<td>38.0 56.5</td>
</tr>
<tr>
<td>Eritrea</td>
<td>51.5 53.3</td>
<td>52.4 55.7</td>
<td>55.6 60.7</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>44.5 50.6</td>
<td>43.3 53.0</td>
<td>46.9 58.1</td>
</tr>
<tr>
<td>Gabon</td>
<td>52.4 55.5</td>
<td>52.9 57.5</td>
<td>55.9 61.5</td>
</tr>
<tr>
<td>Gambia</td>
<td>45.4 47.0</td>
<td>47.1 49.0</td>
<td>51.2 53.0</td>
</tr>
<tr>
<td>Ghana</td>
<td>44.1 60.0</td>
<td>57.2 62.0</td>
<td>61.5 66.0</td>
</tr>
<tr>
<td>Guinea Bissau</td>
<td>44.1 45.7</td>
<td>45.4 47.8</td>
<td>49.2 51.8</td>
</tr>
<tr>
<td>Kenya</td>
<td>52.2 63.6</td>
<td>49.3 65.9</td>
<td>51.5 69.8</td>
</tr>
<tr>
<td>Lesotho</td>
<td>51.2 61.4</td>
<td>40.2 63.7</td>
<td>37.8 67.9</td>
</tr>
<tr>
<td>Liberia</td>
<td>48.1 51.8</td>
<td>55.6 59.3</td>
<td>60.7 63.6</td>
</tr>
<tr>
<td>Malawi</td>
<td>40.7 51.2</td>
<td>39.3 53.2</td>
<td>43.1 57.3</td>
</tr>
<tr>
<td>Mali</td>
<td>50.9 52.6</td>
<td>52.1 54.7</td>
<td>56.1 58.9</td>
</tr>
<tr>
<td>Mozambique</td>
<td>40.6 47.0</td>
<td>38.0 49.0</td>
<td>41.0 53.0</td>
</tr>
<tr>
<td>Namibia</td>
<td>45.1 62.1</td>
<td>44.3 64.5</td>
<td>53.6 68.6</td>
</tr>
<tr>
<td>Nigeria</td>
<td>51.3 55.6</td>
<td>52.1 58.1</td>
<td>55.9 63.1</td>
</tr>
<tr>
<td>Rwanda</td>
<td>39.4 49.2</td>
<td>40.9 50.6</td>
<td>46.5 54.8</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>37.3 39.0</td>
<td>40.5 43.0</td>
<td>44.5 47.0</td>
</tr>
<tr>
<td>S. Africa</td>
<td>56.7 63.3</td>
<td>47.4 65.8</td>
<td>42.0 69.6</td>
</tr>
<tr>
<td>Swaziland</td>
<td>50.8 60.2</td>
<td>38.1 62.7</td>
<td>39.2 67.2</td>
</tr>
<tr>
<td>Togo</td>
<td>51.3 57.1</td>
<td>52.2 59.2</td>
<td>57.0 63.3</td>
</tr>
<tr>
<td>Uganda</td>
<td>41.9 52.0</td>
<td>46.0 54.2</td>
<td>54.3 58.3</td>
</tr>
<tr>
<td>Tanzania</td>
<td>51.1 57.1</td>
<td>52.2 59.2</td>
<td>57.0 63.3</td>
</tr>
<tr>
<td>Zambia</td>
<td>40.5 57.6</td>
<td>42.2 59.6</td>
<td>52.1 63.6</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>42.9 66.5</td>
<td>42.9 68.5</td>
<td>50.2 71.4</td>
</tr>
<tr>
<td>All 35 countries</td>
<td>48.3 54.8</td>
<td>48.2 57.1</td>
<td>52.4 61.2</td>
</tr>
</tbody>
</table>
increases absenteeism and early retirement. Jackson and Pitts (1991), interviewing business holders in Zimbabwe, report that main concerns about HIV/AIDS effects are: loss of skilled labor (33%), loss of manpower (24%), reduced future productivity (24%), insurance and pension costs (13%) and other economic costs (12%). The main effect of the disease on the labor force is absenteeism, since workers becoming ill take time off for medical visits. Morris, Burdge and Cheevers (2001) found that in a South African sugar company, employees who die due to HIV/AIDS take an average of 27.7 additional paid sick days in each of the two years before death, for a total of 55.5 days. More, ING Barings (1999) estimate that a typical employee of South African company loses about 250 productive days over the course of the illness. In East Africa, absenteeism accounts for between a quarter and a half of costs that businesses companies are facing (ILO, 2000a). As a result, in South Africa alone, the epidemic will result in a GDP in 2010 that is 17% lower than what it had been without the virus - the equivalent cost of 0.3-0.4% of annual economic growth.

HIV/AIDS and public finance. HIV/AIDS has also significant impact on the health sector and pension schemes in Africa. For the health sector, handling the epidemic and related diseases constitutes a huge burden. As the demand for HIV-related cure increases, health spending is exploding, far beyond the capacities of the fund providers of the sector (government, health insurance schemes, households, etc.). For instance by the mid-90s, 66 percent of health spending in Rwanda were devoted to treat HIV-related diseases while in Zimbabwe that proportion went over a quarter (ILO, 2000a).

A two-tier economic structure. African economies broadly consist of two sectors, a formal one and an informal one. The formal sector mainly consists of the public sector, parastatal companies and private and modern firms and covers a small part of the labor force. Except for a few countries (Mauritius: 61%, Botswana: 52% and South Africa: 42%), employment in this sector represents less than 20 percent of the labor in Sub Saharan Africa, in many countries the formal sector accounts for less than 10 percent of the labor force as in Gambia: 7%, Ghana: 3%, Cote d'Ivoire: 9%, etc (Gillion et al., 2000).

The reasons for such a two-tier economic structure are often due to institutional barriers: agents often may lack the resources and knowledge required to comply with (in cases) excessive regulations and rules imposed on the formal sector. Costs of formally doing business may be considered as prohibitively expensive, lack of transparency in the legislative processes, corruption, complicated taxation system, lack of confidence and excessively bureaucratic procedures are all possible reasons of why some agents decide to operate outside of the formal sector.

Thus, workers in the formal sector usually benefit from a regular salary set on the basis of a minimum wage, which implies that formal sector wages are not flexible downward. By contrast, the informal sector is considerably larger in developing countries (table 2 suggests that the larger the informal sector, the poorer the economy is). It is characterized (Tokman, 1990; Blunch et al., 2001) by small scales activities with low resources-base, family ownership, labor-intensive, few capital requirements, low level and adapted technology, a simple division of labor and little differentiation in the ownership of means of production. In most African countries, it contributes significantly to output and employment. The informal sector also constitutes a main source

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Charmes (1990, 2000) found out that in Sub Saharan Africa (excluding South Africa), the informal sector contributes between 20 and 50 percent of non-agricultural GDP. In Ghana and Niger, it represents 58 percent of the non-agricultural GDP, in Chad and Mozambique 45 percent and in Senegal 41 percent.
Importance of the informal sector vis-a-vis the formal sector

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>41</td>
</tr>
<tr>
<td>Bolivia</td>
<td>58.2</td>
</tr>
<tr>
<td>Chile</td>
<td>44.7</td>
</tr>
<tr>
<td>Mexico</td>
<td>54.0</td>
</tr>
<tr>
<td>Ivory Coast</td>
<td>53</td>
</tr>
<tr>
<td>Peru</td>
<td>51</td>
</tr>
<tr>
<td>Tanzania</td>
<td>67</td>
</tr>
</tbody>
</table>

Source: Key Indicators of the Labour Market, ILO, 1999

of employment in these countries (between 60 and 80 percent of non-agricultural employment). Employment in this sector consists of two types: non-wage and wage. Non-wage employment represents 80 percent of informal employment in Africa (Charmes, 1990) and consists of self-employment, family labor and apprenticeships. By contrast, wage employment accounts only for about 10 percent and is more casual than regular. Thus e.g. Charmes (1990) reports that in Cameroon, 34 percent of the informal workers are self-employed and 54 percent are apprentices and family workers; non-wage earners represent 88 percent and wage earners for 12 percent of the labour force.

Usually earnings in the informal sector are lower than what salaries are in the formal sector. Moreover, informal sector wages are not legislated as in the formal sector (absence of minimum wage). In South Africa, mean monthly income of African workers in the urban informal sector is less than half of the mean salary paid in the formal one (1,011 versus 2,204 Rand) while non-urban informal workers earn on average 705 Rand (Naledi (2000)). Rama (1998), reporting series of studies comparing earnings in the two sectors, points to a positive earnings gaps (in favor of the formal sector) even in countries with relatively flexible labour markets. However, he sustains that such gaps seldom exceed 30 percent even in countries with large distortions created by labour market policies - except for CFA countries where, by contrast, earnings gaps of 60 percent or more are not uncommon.

Social security. Duality concerns also access to formal social protection. With very few exceptions (Mauritius, Botswana and South African in some extent) formal social security institutions are not broad-based. Pension coverage is low, both in terms of contributors and beneficiaries compared to the population over 60. The ILO (2000b) reports that in 1996 only 0.5 percent of the total population of Benin was covered for an old age pension benefit, with 4.8 percent of the labor force contributing to the scheme. In Mauritania these percentages were 1.5 and 3.3 and in Uganda 4.1 and 8.2.

Except for a few countries (Ghana 17%, Equatorial Guinea 26%), pension contributions are rather low in Sub Saharan Africa. Standing on average at less than 10 percent however, pensions paid in counterpart of this contribution can be quite generous, depending of the occupational sector. In some countries, civil servants’ pensions correspond to more than 60 percent of the average wage (80 percent in Cote d’Ivoire, 75 in Senegal, 66 in Cape Vert). However, the average replacement rate (public and private) in Africa at stands at around 50 percent or less (see table 4).
Table 3
Retirement systems in a selection of countries. Importance and coverage

<table>
<thead>
<tr>
<th>Country</th>
<th>Importance as % of GDP</th>
<th>Rate of coverage (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bangladesh</td>
<td>0.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Niger</td>
<td>0.2</td>
<td>2.8</td>
</tr>
<tr>
<td>Chad</td>
<td>0.0</td>
<td>1.1</td>
</tr>
<tr>
<td>Mali</td>
<td>0.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Mozambique</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>U.S.</td>
<td>6.5</td>
<td>68</td>
</tr>
<tr>
<td>Denmark</td>
<td>9.9</td>
<td>100</td>
</tr>
<tr>
<td>U.K.</td>
<td>9.5</td>
<td>94.2</td>
</tr>
</tbody>
</table>


Table 4: Percentage of recent average earnings paid as pension after thirty years of covered employment, 1992

<table>
<thead>
<tr>
<th>Countries</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benin</td>
<td>60</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>40</td>
</tr>
<tr>
<td>Burundi</td>
<td>50</td>
</tr>
<tr>
<td>Cameroon</td>
<td>45</td>
</tr>
<tr>
<td>Central Africa Rep.</td>
<td>45</td>
</tr>
<tr>
<td>Chad</td>
<td>48</td>
</tr>
<tr>
<td>Congo</td>
<td>50</td>
</tr>
<tr>
<td>Gabon</td>
<td>45</td>
</tr>
<tr>
<td>Liberia</td>
<td>51</td>
</tr>
<tr>
<td>Mauritania</td>
<td>40</td>
</tr>
<tr>
<td>Niger</td>
<td>40</td>
</tr>
<tr>
<td>Rwanda</td>
<td>45</td>
</tr>
<tr>
<td>Sudan</td>
<td>50</td>
</tr>
<tr>
<td>Togo</td>
<td>40</td>
</tr>
</tbody>
</table>
Private old-age support. The majority of the African population relies on informal arrangements (community or family based) for their old age support, especially in the informal sector where agents rely on a micro-support system where they are supported by their offspring and/or their local communities. World Bank (1994) indicates that in Nigeria, more than 97 percent of the urban old and 93 percent of the rural old report receiving some financial or material support from family or kin. In Kenya, 88 percent of persons over 65 to receive income from their family, according to a survey conducted in Demo. Rep. of Congo (Kalasa, 1997), 66 percent of active workers report that they expect support from their offspring once they retire. This contrasts with the situation in OECD countries where 100 percent of persons over 65 receive income from pension and welfare schemes.

Transfers to aged constitute one aspect of inter and intrafamily transfers (in cash or in kind) in Africa. However, reliable and sound data on the share of active income transferred to the aged is scant: based on the Senegalese household expenditure-income survey (Esam, 1995) one can estimate that the share stands at around 5 percent or slightly above. The survey reports that 19 percent of the income received by households are transfers, and 27 percent of transfers received by households go to the aged.

Population growth. There is little doubt that introducing a wide-spread old-age income support scheme would be desirable from an ethical point of view. Although the birth rate in developing countries is high, at least relative to developed countries, the population is nevertheless forecast to age significantly. Currently it is estimated that the older population comprises slightly over 38 million today in Africa. By 2050 this figure is however projected to reach 212 million - a six-fold increase in five decades, even despite HIV/AIDS. Hence, it seems clear that the rise in the number of ‘older’ people provides a challenge both to individual countries as well as to the continent as a whole.

3 The model

Our model depicts a small open developing economy with individuals, firms and the government. We distinguish between healthy and unhealthy individuals and assume that individuals do not save and that capital exclusively comes from abroad. Firms operate in the formal sector or in the informal sector. The government consists provides a mandatory public pension scheme whose coverage is restricted to the formal sector workers.

3.1 Formal and informal sectors

In the formal sector, a composite good at time \( t \) (\( Y_t \)) is produced by a representative firm which uses capital (\( K_t \)) and labor, measured in efficiency units (\( L_t \)). The production function

\[
Y_t = F(K_t, L_t)
\]

exhibits constant returns to scale so that output per efficiency unit of labor (\( y_t = \frac{Y_t}{L_t} \)) may be expressed as a function of the capital stock per efficient worker (\( k_t = \frac{K_t}{L_t} \)): \( y_t = f(k_t) \). The representative firm behaves competitively on the factor markets, equalizing the marginal productivity

\(^5\text{E.g. in 'tontines' in West Africa, co-operative arrangements exist to help provide medicaments and medical services as informal social security arrangements.}\)
of each factor to its rate of return. Let us denote the marginal productivity functions as

\[ \rho(k_t) = f^0(k_t) \]  

(2)

and

\[ \omega(k_t) = f(k_t) \cdot k_t \rho(k_t). \]  

(3)

We consider a small open economy case where all capital is provided by foreign investors. Foreign investments in the country are such that the domestic interest rate equals the international interest rate plus a time variable risk premium \( \pi \) related to domestic factors such as political instability and country risks of insolvency. The exogenous rate of return on capital \( \rho \) signifies the domestic level of capital per efficiency unit of labor which, in turn, signifies the equilibrium gross and net wage rates:

\[ k^n = \rho^{-1}(1 + r^n + \pi) = \rho^{-1}(R) \]

\[ w = w(k^n) \]

where \( R \) is the equilibrium factor of interest.

Hence, perfect mobility of financial assets enables us to consider that both the interest rate and the net wage rate are constant over time; they are independent of domestic savings and labor supply. In this case, all agents within each group opt for the same decision.

The informal sector is modelled in a simple way; firms consist in self-employment activities giving rise to an exogenous and constant wage, \( b \). No capital is used here and the wage rate is independent of the number of laborers in this sector.

3.2 Individuals

The economy we are considering consists of a large number of agents. These agents are distinguished in the sense that a certain proportion of the population is healthy (denoted by \( h \)). The remaining proportion is unhealthy (denoted by \( u \)), i.e. infected by HIV/AIDS. Agents that remain healthy throughout their lives live for three periods (young, adult and old). The number of adults at time \( t \) is denoted by \( N_t \).

There is the risk of becoming infected by HIV/AIDS, both when young and when adult. When infected, agents die at the end of the period. This means that only an exogenous proportion of children born at time \( t \) will make it to adulthood. The remaining proportion is infected and will not reach adulthood. More precisely, each adult gives birth to \( 1 + n \) children. For simplicity, we consider that at least one of these children is healthy and will make it to adulthood. The remaining \( n \) children at time \( t \) are all healthy with a probability \( Q_t \), or all unhealthy with a probability \( 1 - Q_t \). This simplifying assumption enables to model the effect of HIV/AIDS on uncertainty about the number of descendents and on the population growth rate. Obviously, we have:

\[ N_{t+1} = N_t(1 + nQ_t) \]  

(4)

Similarly, an exogenous proportion \( P_t \) of adult at time \( t \) will make it to old-age. The remaining proportion \( 1 - P_t \) will die at the end of the period.
Each adult must choose between working in the formal or in the informal sector. They do so by maximizing a utility function depending on the levels on consumption when adult and when old (only when adult if they are infected). There is no saving so that the only decision is the choice of the sector in which labor is supplied. We use a logarithmic utility function, which for a healthy adult is given by

\[
U_t = \ln(c_t) + \ln(d_{t+1}).
\]

The utility function of an infected adult is simply given by \(U_t = \ln(c_t)\).

Adult workers in the formal sector participate in a mandatory public pension scheme and pay a proportional tax on wages (at the rate \(\tau_t\)). When old they expect to receive a pension benefit. It is broadly documented that in developing countries, agents find public institutions unpredictable and unreliable. Therefore, one could assume that they do not fully trust public announcements. To formalize this, we assume that agents raised in the formal sector trust the government announcement and expect to receive \(b_{t+1}\) when old. On the contrary, agents involved in familial arrangements find it risky to join the public sector. More precisely, individuals raised in the informal sector by a healthy parent have a limited trust in government promises and expect to receive \(\theta b_{t+1}\). The parameter \(0 < \epsilon < 1\) then captures the level of trust in government announcement.

There is no collective pension scheme in the informal sector. However, within each family, aged persons are looked after by adults through an informal social security scheme: each working adult supports the old age generation by transferring resources to the latter (the amount transferred by each surviving adult to his parent is denoted by \(v < b\)). An infected worker cannot supply the same number of hours as a healthy worker. We denote by \(\theta < 1\) the relative labor supply of unhealthy workers. Absenteeism related to HIV/AIDS is then captured in \(1-\theta\).

Agents are heterogenous in two characteristics, their health status (\(h\) or \(u\)) and their obligations to transfer resources: \(s\) denotes the obligation to support parents and \(n\) denotes the absence of such an obligation. As shown in the figure, the latter type depends on the sector in which individuals were raised as well as on their parent’s health status. An obligation to transfer resources is observed for adults raised in the informal sector by a healthy parent.

For each type of agent, decisions at time \(t\) can be written as follows.

For healthy adult who do not have to support their parents (\(h,n\)), the decision program is

\[
\rho_t(h, n) : \ln(b) + Q_t \ln[(1 + n)v] + (1 - Q_t) \ln[v] + \ln[w(1 - \tau_t)] + \ln[b_{t+1}]
\]

Opting for the formal sector is optimal if

\[
\tau_t \cdot \ln \left( \frac{v(1 + n)Q_t}{w} \right) \geq \theta \ln \left( b_{t+1} \right)
\]

For unhealthy adult who do not have to support their parents (\(u,n\)), the decision program is

\[
\rho_t(u, n) : \ln[\theta b] + \ln[\theta w(1 - \tau_t)]
\]

Note that unhealthy workers do not expect to survive for the next period and thus have not to anticipate the future pension claim.
Opting for the formal sector is optimal if

$$\tau_i \cdot 1 \cdot \frac{b_i}{w} \cdot \tau^u$$

(6)

For healthy adult who have to support their parents (h,s), the decision program is

$$\rho_i(h,s) : \ln(b_i \cdot v) + Q_i \ln((1+n)v) + (1_i \cdot Q_i) \ln[v] \cdot \ln[w(1_i \cdot \tau_i) \cdot v] + \ln[eb_{i+1}]$$

Opting for the formal sector is optimal if

$$\tau_i \cdot 1 \cdot \frac{v}{w} \cdot \frac{(b_i \cdot v)(1+n)Q_i}{eb_{i+1}} \cdot \tau^{hs}(Q_i, b_{i+1})$$

(7)

For unhealthy adult who have to support their parents (u,s), the decision program is

$$\rho_i(u,s) : \ln(b_i \cdot v) \cdot \ln[w(1_i \cdot \tau_i) \cdot v]$$

As for other unhealthy agents, opting for the formal sector is optimal if

$$\tau_i \cdot \tau^u$$

Aggregating these decisions, the labor supply (in efficiency units) in the formal sector can be written as

$$L_i = N_i P_i \pi^h_i + N_i (1 \cdot P_i) \theta \pi^u_i$$

(8)

where $\pi^h_i$ and $\pi^u_i$ denote the share of healthy and unhealthy adults opting for the formal sector. These shares are endogenously obtained by comparing the rate of tax to be paid in the formal sector ($\tau_i$) to the critical values $\tau^{hn}(Q_i, b_{i+1}), \tau^{hs}(Q_i, b_{i+1})$ and $\tau^u$.

We derive the following lemma.
Lemma 1 When \( b_{t+1} > v(1 + n)Q_t \), the critical value of tax such that unhealthy workers join the formal sector is lower than the one such that healthy who have no parents to support join: 
\[
\tau^u < \tau^{hn}(Q_t, b_{t+1})
\]

Proof. Obviously comes from the definition of \( \tau^{hn}(Q_t, b_{t+1}) \) and \( \tau^u \). ■

Lemma 2 For any \( \varepsilon \in [0, 1] \) such that \( \varepsilon b_{t+1} < v(1 + n)Q \), the critical value of tax such that unhealthy workers join the formal sector is higher than the one for healthy workers who have to transfer resources to their parents: 
\[
\tau^{hs}(Q_t, b_{t+1}) < \tau^u
\]

Proof. Obviously comes from the definition of \( \tau^u \) and \( \tau^{hs}(Q_t, b_{t+1}) \). ■

These two lemma induce that, if both conditions hold \( \varepsilon b_{t+1} < v(1 + n)Q < b_{t+1} \), agents are more willing to join the formal sector if they do not have to support their parents, i.e. when there is no family arrangement for old-age support or when this arrangement is broken (due to HIV/AIDS). Alternatively, if an agent is involved in a private old-age support system, she is more reluctant to join the formal sector and pay taxes. The intuition behind the first assumption seems clear, given that informal social security schemes are assumed not to exist in the formal sector: if agents have to support their parents through the informal system and decide themselves to migrate to the formal sector, they are in effect paying two contributions, \( v \) and \( \tau_w \), however only receive a return on the latter in the form of a PAYG scheme. Hence, they are less willing to migrate to the formal sector than agents who do not have to support any parents.

3.3 Social security

In the formal sector, social security operates as a PAYG system. Taxes paid by workers in the formal sector are used to finance benefits of surviving retirees. The equilibrium amount of taxes paid by workers is given by

\[
\tau_t = \frac{\varepsilon}{w_t} \frac{b_t \pi^h_{t+1} P_{t+1} - \frac{1}{1 + nQ_{t+1}}}{P_t \pi^h_t + (1 - P_t)\theta \pi^h_t [1 + nQ_{t+1}]}
\]

Obviously, the number of beneficiaries is given by healthy workers in the formal sector at the previous period \( (\pi^h_{t+1} P_{t+1} N_{t+1}) \). Unhealthy workers die at the end of adulthood and thus cannot receive their pension claim. The number of contributors is given by (8), i.e. the sum of healthy and unhealthy workers in the formal sector \( (N_t P_t \pi^h_t + N_t (1 - P_t)\theta \pi^h_t) \).

3.4 Equilibrium types

A "hn-equilibrium" \( (\tau^u < \tau_t \cdot \tau^{hn}(Q_t, b_{t+1})) \) is obtained when the equilibrium amount of tax in the formal sector is such that all unhealthy workers opt for the informal sector whilst a proportion of healthy workers opt for the formal sector (those who do not have to support their parent). In that case, we have

\[
\pi^u_t = 0 \quad \pi^h_t = P_{t+1} \pi^h_{t+1} \frac{1}{1 - P_t} + P_t \pi^h_t
\]
The proportion $\pi^h$ if the ratio of the sum of adults raised in the formal sector and those raised in the informal sector by an infected parent, expressed as a percentage of the healthy population at time $t$. The numerator amounts to $P_t \frac{1}{i} N_{ti} \frac{1}{i} \pi^h + (1 \frac{1}{i} P_t \frac{1}{i} 1 + n Q_{ti} \frac{1}{i} 1)$ and the denominator is simply $N_t P_t$. The proportion is obtained after simplifications.

Using (9), such an equilibrium holds when

$$\tau^u < \frac{\mathcal{F} \frac{b_t \pi^h}{\pi^h P_t \frac{1}{i} 1 + 1 \frac{1}{i} P_t \frac{1}{i} 1 [1 + n Q_{ti} \frac{1}{i} 1]}}{\frac{w}{w}} \cdot \tau^{hn}(Q_t, b_{t+1})$$

where $\tau^u$ is given in (6) and (5) respectively. If the right condition does not hold, all (healthy and unhealthy) agents opt for the informal sector. Such a "0-equilibrium" ($\tau^{hn}(Q_t, b_{t+1}) < \tau_t$) is characterized by

$$\pi_t^u = 0$$
$$\pi_t^h = 0$$

and is obtained when

$$\frac{\mathcal{F} \frac{b_t \pi^h}{\pi^h P_t \frac{1}{i} 1 + 1 \frac{1}{i} P_t \frac{1}{i} 1 [1 + n Q_{ti} \frac{1}{i} 1]}}{\frac{w}{w}} > \tau^{hn}(Q_t, b_{t+1})$$

A "u-equilibrium" ($\tau^{hs}(Q_t, b_{t+1}) < \tau_t \cdot \tau^u$) is obtained when the equilibrium amount of tax in the formal sector is such that all unhealthy workers opt for the formal sector whilst a proportion of the healthy workers (those who do have to support their parents) opt for the informal sector. In that case, we have

$$\pi_t^u = 1$$
$$\pi_t^h = P_t \frac{1}{i} 1 \pi^h + 1 \frac{1}{i} P_t \frac{1}{i} 1$$

Hence, using (8) and (9), such an equilibrium holds when

$$\tau^{hs}(Q_t, b_{t+1}) < \frac{\mathcal{F} \frac{b_t \pi^h}{\pi^h P_t \frac{1}{i} 1 + 1 \frac{1}{i} P_t \frac{1}{i} 1 [1 + n Q_{ti} \frac{1}{i} 1]}}{\frac{w}{w}} \cdot \tau^u$$

Finally, a "hs-equilibrium" is ($\tau_t < \tau^{hs}(Q_t, b_{t+1})$) is obtained when the equilibrium amount of tax in the formal sector is such that all workers opt for the formal sector. In that case, we have

$$\pi_t^u = 1$$
$$\pi_t^h = 1$$

Using (5-7), (8) and (9), such an equilibrium holds when

$$\frac{b_t \pi^h}{w[P_t + \frac{1}{i} P_t 1 [1 + n Q_{ti} \frac{1}{i} 1]]} \cdot \tau^{hs}(Q_t, b_{t+1})$$

Under the conditions of lemma 1 and lemma 2 ($e b_{t+1} < v(1 + n)^Q < b_{t+1}$), we have $\tau^{hs}(Q_t, b_{t+1}) < \tau^u < \tau^{hn}(Q_t, b_{t+1})$. Hence, as depicted on figure 2, the effective tax rate given by (9) determines the equilibrium type of our economy.
Thus, following table gives the proportions of healthy and unhealthy agents in the various equilibria:

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>HS</th>
<th>U</th>
<th>HN</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^u$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^h$</td>
<td>$P_{i_1}$</td>
<td>$P_{i_1}$</td>
<td>$P_{i_1}$</td>
<td>$P_{i_1}$</td>
</tr>
</tbody>
</table>

3.5 Initial conditions

The purpose of our model is to examine the impact of an HIV/AIDS shock on the endogenous proportion of workers joining the formal sector. Consider that the shock occurs at time 1. Before that date, there is no agent infected so that $P_t = Q_t = 1$. As highlighted in section 2, most developing countries are characterized by a low share of workers in the formal sector. To be consistent with this evidence, we require our initial solution to be of a hn-type.

It means that there is no mobility between the formal and the informal sectors. More precisely, the initial tax rate (at time $0$ and before) must such that

1. individuals raised in the formal sector have no incentive to leave this sector, i.e. $\tau_0 = \frac{b_0}{w_0(1+n)} < \tau^{hn}(1, b_1)$;
2. individuals raised in the informal sector have no incentive to join the formal sector, i.e. $\tau_0 = \frac{b_0\pi^h}{w_0(1+n)} > \tau^{hs}(1, b_1)$.

To simplify notation, let us consider a pension scheme with de ned bene t so that $b_t$ is constant over time ($b_t = b$ $t$) and $w_t = 1$. In that case, we obtain the following lemma:

Lemma 3 When $P = Q = 1$, an hn equilibrium is obtained when $1_{1} < \frac{\Phi_i(1+n)}{e^b} \frac{1}{\pi^0}$

Proof. Obviously comes from the two previous conditions.

If the condition of lemma 3 holds, our initial economy is such that the share of workers in the formal sector is stable over time: $\pi_{i}^{h} = \pi_{i}^{h}$ $t = 0$. Let us examine how a HIV/AIDS shock influences the proportion of workers opting for the formal sector. We distinguish two cases, the case of a permanent shock and the case of a temporary shock.
4 Social security & permanent HIV-shock

The critical values of tax only depend on the proportion of infected workers and infected children, i.e. on \( P_t \) and \( Q_t \). A shock to these variables is likely to modify the endogenous proportions of healthy and unhealthy workers opting for the formal sector. In this section we analyze the effect of a permanent shock. As discussed above, there is no agent infected before time 1: \( P_0 = Q_0 = 1 \). At time 1, the proportions of healthy adults and children individuals permanently fall to \( P < 1 \) and \( Q < 1 \). These values characterize the new long-run state of the economy. Of course, there is no reason to have \( P = Q \). Data on HIV/AIDS from African countries reveal that adults' prevalence rates are higher than children's prevalence rates (figure 1 above), thus the realistic case is represented in the next figure.

![Fig. 3 - Permanent shock on P and Q](image)

How does this shock affect our endogenous sharing of the population? We distinguish two periods and derive the long-run implications of the shock. At time 1, HIV/AIDS only affect young and adult workers but not the elderly. From time 2 onwards, all generations can be infected.

At time 1, HIV/AIDS exerts an ambiguous effect on the equilibrium tax rate. On the one hand it reduces the average labor supply of workers in the formal sector since a proportion of them becomes unhealthy (labor supply decreases due to absenteeism: \( \theta < 1 \)). On the other hand, following lemma 1 and 2, the shock reduces the attractiveness of the private old-age support system and can result in migration from the informal to the formal sector (hence, increasing the number of contributors). Consequently, at time 1, the HIV/AIDS shock only affects the income side of the social security scheme.

For ambiguous cases, our analysis relies on numerical simulations. We use a parameter set consistent with the stylized facts of section 2:

1. the wage rate in the informal sector amounts to 75 percent of formal sector wage \( (b/w = .75) \);
2. in the informal sector each adult transfers 30 percent of his wage to his surviving parent \( (v/b = 0.3) \).
3. in the formal sector, the social security replacement rate amounts to 80 percent \( (b/w = .8) \);
the relative labor supply of unhealthy worker, \( \theta \), equals 70 percent;

2 population growth is determined by \( n = .7 \) (which implies an annual population growth of about 2.7 percent);

2 finally, the rate on trust in the public pension scheme amounts to 40 percent (\( \epsilon = .4 \)) and the share of adults initially working in the formal sector equals 10 percent (\( \pi^0_0 = .1 \)).

This parameter set veriﬁes the condition in lemma 3: a "hn-equilibrium" emerges at the date 0. Given the calculations in section 3.4, the type of equilibrium is fully determined by the combination \((P,Q)\) of prevalence rates. The following set of conditions characterizes the situation at time 1:

2 a "hs-equilibrium" emerges if
\[
P \cdot \frac{3}{4} \frac{h_n^b}{1 + nQ} \frac{1}{\tau^{hs}} \theta i \frac{1}{1 + \theta i \tau};
\]

2 a "u-equilibrium" emerges if
\[
P \cdot \frac{1}{\sigma_i + h_0} \theta i \frac{b_n h}{1 + n \theta i \tau};
\]

2 a "hn-equilibrium" emerges if
\[
P \cdot \frac{h}{\tau^{hn}(1 + n)};
\]

2 a "0-equilibrium" emerges if the above conditions do not hold.

Each of these conditions can be represented as a locus in the plan \((P,Q)\). The critical curves are represented in .gure 4 (case 1). According to our parameter set, migration from the informal to the formal sector for healthy individuals (i.e. the hs critical line) is mainly aected by the prevalence rate of children. If \( Q \) falls below approximately 90 percent, the return of the private old-age support system falls below the expected return on the public pension scheme and migration is likely to occur. On the contrary, the decision of unhealthy workers (i.e. the uu critical line) mainly depends on the prevalence rate of adults, \( P \). This variable exerts an ambiguous in‡uence on the equilibrium. In accordance with conventional wisdom, HIV/AIDS reduces the e¤ective labor supply in the formal sector and thus pushes the contribution rate upwards. This can lead to an "0-equilibrium" if the proportion of infected workers increases. Nevertheless, if \( P \) falls su¢ciently, the number of unhealthy becomes very large, which in turn exerts a strong impact on the tax rate. Indeed, given \( \tau^{hs} < \tau^u \), unhealthy workers are more willing to migrate to the formal sector. Migration from the informal to the formal sector can be observed since unhealthy adults raised in the informal sector are now only concerned with the net wages in both sectors. According to our simulations, if \( P \) falls below approximately 85 percent, migration of unhealthy workers to the formal sector reduces the equilibrium tax rate and a "u-equilibrium" occurs.

At time \( t = 2 \) the e¤ect of HIV/AIDS is also ambiguous. The only difference to time 1 is that HIV/AIDS also aects the old-age population (the number of pensioners decreases). In that case, the shock impacts on both the income and expedenditure sides of the public pension scheme. Assuming that the shock does not modify the type of equilibrium at time 1 (i.e. a hn equilibrium occurs at time 1), it follows that \( \pi^1_i = \pi^0_i \). The following set of equations characterizes the situation at time 2:

2 a "hs-equilibrium" emerges if
\[
P \cdot \frac{1}{1 + nQ} \frac{\bar{C}}{\theta} \frac{h_n h}{\tau^{hs}} i \frac{1}{(1 + nQ)(1 + \theta i \tau)};
\]
a "u-equilibrium" emerges if
\[ h (1 + nQ)(1 + \theta) P^2 + \frac{h \theta_{0} \theta}{(1 + nQ)(1 + i \theta)} P_i (1 + nQ) \theta < 0; \]
a "hn-equilibrium" emerges if
\[ \frac{h}{1 \pi_{0}^{i}} (1 + \frac{b h_{0}}{(1 + nQ)(1 + \mu i \theta)} P_i )^{i} < \frac{b h_{0}}{1 + nQ} ; \]
a "0-equilibrium" emerges if the above conditions do not hold.

The critical curves are represented on figure 4 (case 2). We obtain a similar figure as for time 1 except that the "hs-locus" is now negatively sloped. It is worth noting that the possibility of an "0-equilibrium" solution is ruled out. Indeed, following the above definition of the hn-locus, it would require at least \( \tau_{hn}(Q, b) < \frac{b}{1 + nQ} \). Such a possibility is reasonably ruled out by our parameter set. Hence, the number of adults working in the formal sector is always positive at time 2.

Let us finally investigate the long-run effect of a permanent HIV/AIDS shock. If a "hs-equilibrium" emerges at time 2, the proportion of healthy workers in the formal sector jumps to one. In the more reasonable case where a "hn-equilibrium" or "u-equilibrium" emerges at time 2, the size of the formal sector progressively increases over time, as depicted in the following proposition:

**Proposition 1** If \( \tau_{hn}(Q, b) > \frac{b}{1 + nQ} \) (a "0-equilibrium" is ruled out) and if a "u-equilibrium" or a "hn-equilibrium" emerges at time 2, a permanent shock in P and Q has two implications: (i) the proportion of healthy workers joining the formal sector converges to one and (ii) during that transition, the contribution rate progressively increases over time.

**Proof.** At the "hn-equilibrium" or "u-equilibrium", we have \( \pi_{h}^{\pi} = \pi_{h}^{\pi} \pi_{P} + 1 \pi_{P} > \pi_{h}^{\pi} \pi_{P} \) (at least for \( P < 1 \)). The proportion \( \pi_{h}^{\pi} \) thus converges to its maximal value \( \pi_{h}^{\pi} = 1 \). On the transition, using (9), we have \[ \frac{\partial \pi_{h}^{\pi}}{\partial \pi_{h}^{\pi}} = \frac{b h_{0}}{(1 + nQ)(1 + i \theta)} \left[ \frac{\theta_{0} \theta P + P_{1} P + (1 + i \theta) \theta \pi_{h}^{\pi}}{\pi_{h}^{\pi} P_{1} + P_{1} P_{0} + (1 + i \theta) \theta \pi_{h}^{\pi}} \right] > 0. \] The equilibrium tax rate progressively increases over time. As before, the condition \( \tau_{hn}(Q, b) > \frac{b}{1 + nQ} \) ensures that the economy cannot jump to an "0-equilibrium".

The dynamic process leads to a continuous increase in the proportion of healthy workers opting for the formal sector and a continuous increase in the contribution rate. A permanent shock progressively breaks family ties and makes the private system vanish over time. Some healthy workers in the informal sector lose their parent and are more willing to trust the public pension scheme. Hence they and an incentive to join the formal sector. Therefore, the formal social security system becomes the only source of protection for old-age individuals. Over that transition, the number of pensioners increases and as the public pension scheme gains in maturity, the equilibrium tax rates progressively increases.
Fig. 4 - Permanent shock and equilibrium type

Case 1: situation at time 1

Case 2: situation at time 2 (with $\pi^{A}_{H} = \pi^{B}_{H} = .10$)
5 Social security & temporary HIV-shock

Let us now analyze the effect of a temporary shock. Before time 1, no agent is infected: $P_0 = Q_0 = 1$. At time 1, the proportions of healthy and unhealthy individuals fall to $P < 1$ and $Q < 1$. Then, at time 2, the mortality rates go back to their initial values. This case is represented on the next figure.

Fig. 5 - Temporary shock on P and Q

Obviously, there is now an additional transition period. We must distinguish three periods. At time 1, HIV/AIDS only affects young and adult workers but not the elderly. At time 2, all adults are healthy and the shock only affects the aged. From time 3 onwards, all generations are healthy. The difference with the previous section concerns the proportion of adults in the formal sector.

At time 1, the type of equilibrium is determined by the same set of conditions than for a permanent shock. The effect of the HIV/AIDS shock on the proportion of workers in the formal sector is ambiguous. Figure 4 (case 1) depicts the critical prevalence rates at time 1.

At time 2, all adults are healthy so that only two types of solution can be obtained, a "hs-equilibrium" or a "hn-equilibrium". However, the number of old agents is lower than in the previous periods. Let us assume that the shock does not modify the type of equilibrium at time 1 (i.e. a "hn-equilibrium" occurs at time 1). It follows that $\pi^h_1 = \pi^h_0$. The following set of equations then characterizes the situation at time 2:

1. A "hs-equilibrium" emerges if $P \cdot \frac{r^h(1+n)}{b^h} > \frac{1}{1 + \frac{r^h}{i}}$;
2. A "hn-equilibrium" emerges if $P \cdot \frac{h}{1 + \frac{r^h}{i} + \frac{b^h}{i} + \frac{1}{1 + \frac{r^h}{i}}}$;
3. A "0-equilibrium" emerges if these conditions do not hold.

If the first condition holds, all workers opt for the formal sector. If the above conditions do not hold, a "0-equilibrium" emerges and all workers opt for the informal sector. Between these two extreme cases, a "hn-equilibrium" emerges and we derive the following result:

**Proposition 2** If a "hn-equilibrium" emerges at time 2, the proportion of workers in the formal sector increases
Proof. At the “hn-equilibrium”, we have \( \pi_2^h = P\pi_0^h + 1 \) if \( P > \pi_0^h \).

Intuitively, this result can be interpreted as before. At time 2, some workers raised in the informal sector are willing to join the formal sector due to a break in family ties and greater confidence in public institutions. In the case of a permanent shock, this migration flow makes the share of the formal sector converging to one. In the case of a temporary shock, the flow is one-shot.

For subsequent periods \((t \geq 3)\), all the population members are healthy. According to lemma 3, we have

Proposition 3 If a “hn-equilibrium” occurs at time 0, a “hn-equilibrium” occurs at time 3 and onwards

Proof. The condition for “hn-type” to be a potential equilibrium is the same as at time 0, \( \frac{b_n}{1+n} < \tau^{hn}(1,b) \). The condition for a “hs-type” not to be an equilibrium is less restrictive than at period 0: \( \frac{b_n}{1+n} > \tau^{hs}(1,b_1) \).

A temporary shock thus raises the proportion of workers joining the formal sector at time 2, when the infected generation retires. In subsequent period, this proportion becomes stationary.

6 Conclusion

The above paper has investigated the implications if HIV/AIDS on social security systems modelled on African stylized facts, i.e. capturing a two-tier sector economy combined with a two-tier old-age support framework: formal and informal social security arrangements exist in the formal and informal sector respectively.

Starting from a scenario where there is no HIV/AIDS and only a small percentage of the population/labour force in the formal sector, we show that a permanent decrease in the survival probability of children and adults, such as caused by HIV/AIDS, will lead to migration to the formal sector combined with increasing social security rates. The informal sector (and hence the informal social security system) will cease to exist, however, whilst this could be immediate, it is more likely that it will be a gradual process. In addition, we show that a transitory shock to survival probabilities will increase the proportion of workers in the formal sector, after which it will remain constant. These results are based on assumptions which are relevant to developing economies; in particular, they are based on the fact that

1. that agents are more willing to join the formal sector if they do not have to support their parents,
2. that agents in the informal sector trust public institutions less than agents in the formal sector,
3. that the initial equilibrium in an ‘hn’ type at time \( t = 0 \) and that Lemma 3 holds (i.e. \( \tau^{hs} < \tau < \tau^{hn} \)).

There are several ways this model could be extended. For one, one could introduce capital into the model by assuming that e.g. formal sector agents have access to capital markets and
can hence save and dissave upon retirement, whilst informal sector agents are unable to do so. This would endogenize capital, yet would no doubt lead to a more complex model setup. In addition, one would have to discuss to what extent capital in developing economies originates from domestic sources and to what extent from foreign sources (issues such as capital flight and foreign FDI and aid would play a role here).

A further extension would be to endogenize the population growth rate. In our model we have assumed that it is exogenously given, however, one of the reasons that it is high in developing countries is precisely due to the need for children to provide support in one's old age (i.e. the informal social security system). The occurrence of HIV/AIDS could bring about two effects. On the one hand, migration to the formal sector could lead to a reduction in the population growth rate as those in the formal sector would have less need for more children due to the formal social security scheme. On the other hand, increases in child mortality could also lead to an increase in the reproduction rate as parents can no longer be sure how many of their offspring will survive and hence be able to support them in their old age.

Finally, in this paper we have modeled a PAYG security scheme. Given the problems PAYG is facing in developed countries, one could argue that developing economies would best be advised to opt for a fully funded scheme to avoid pitfalls of the former. Whilst this would be an interesting extension, one has to point to the weak financial sector that is characteristic of undeveloped regions: there is little competition in the financial sector and access for the large majority of the population is limited, thus currently the introduction of any fully funded scheme would have to be preceded by liberalization of the financial sector coupled with institutional reforms.
References


